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Theory of Complex Variables - MA 209
Problem Sheet - 1
Complex Number and Their Properties

1. Evaluate the following powers of i.
   (a) i^8
   (b) i^{105}

2. Write the given numbers in the form of $a + ib$.
   (a) $2i^3 - 3i^2 + 5i$
   (b) $2i^6 + (\frac{2}{-i}) + 5i^{-5} - 12i$
   (c) $(3 + 6i) + (4 - i)(3 + 5i) + (\frac{1}{2-i})$
   (d) $\frac{4+5i+2i^3}{(2+i)^2}$
   (e) $(2 + 3i)(\frac{2-i}{1+2i})^2$
   (f) $\frac{i}{1+i}$

3. Uset binomial theorem, to write the given number in the form $a + ib$.
   (a) $(1 - \frac{1}{2}i)^3$
   (b) $(-2 + 2i)^3$

4. Find Re(z) and Im(z) of the following.
   (a) $(\frac{i}{3-i})(\frac{1}{2+3i})$
   (b) $\frac{1}{(1+i)(1-2i)(1+3i)}$

5. Let $z = x + iy$. Write the following numbers in terms of x and y.
   (a) $Re(1/z)$
   (b) $Re(z^2)$
   (c) $Im(2z + 4\overline{z} - 4i)$
   (d) $Im(\overline{z}^2 + z^2)$

6. Let $z = x + iy$. Write the following numbers in terms of Re(z) and Im(z).
   (a) $Re(iz)$
   (b) $Re(z^2)$
   (c) $Im(iz)$
   (d) $Im((1 + i)z)$

7. Solve the following for real and imaginary parts.
   (a) $2z = i(2 + 9i)$
   (b) $z - 2\overline{z} + 7 - 6i = 0$
   (c) $z + 2\overline{z} = \frac{2-i}{1+3i}$
   (d) $z^2 = i$
   (e) $\overline{z}^2 = i$
   (f) $\frac{z}{1+\overline{z}} = 3 + 4i$

8. What can be said about the complex number z if $z = \overline{z}$? If $(z)^2 = (\overline{z})^2$?

9. Without doing any siginifant work, evaluate $(1 + i)^{5404}$.

10. For n, a nonnegative integer, i^n can be one of four values : $1, i, -1, -i$. In each of the following four cases, express the integer exponent n in terms of the symbol k, where $k = 0, 1, 2, \ldots$
    (a) $i^n = 1$
    (b) $i^n = i$
    (c) $i^n = -1$
    (d) $i^n = -i$.

11. Suppose z_1 and z_2 are complex numbers. What can be said about z_1 or z_2 if $z_1 z_2 = 0$?

12. Suppose the product $z_1 z_2$ of two complex numbers is a nonzero real constant. Show that $z_2 = k\overline{z}_1$, where k is a real number.

13. Prove that $z_1\overline{z}_2 + \overline{z}_1 z_2 = 2Re(z_1 z_2)$.

14. Mathematicians like to prove that certain "things" within a mathematical system are unique. For example, a proof of a proposition such as "The unity in the complex number system is unique" usually starts out with the assumption that there exist two different unities, say, ℓ_1 and ℓ_2 , and then proceeds to show that this assumption leads to some contradiction. Give one contradiction if it is assumed that two different unities exist.

15. Follow the procedure outlined in the above problem to prove the proposition "The zero in the complex number system is unique."

16. Solve the given system of equations for z_1 and z_2:

$$\begin{aligned} -1z_1 + (1+i)z_2 &= 1+2i \\ (2-i)z_1 + 2iz_2 &= 4i. \end{aligned}$$

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